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The charge and mass perturbative renormalization in explicitly covariant LFD

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Abstract. In several preceding studies, the explicitly covariant formulation of light-front dynamics was developed and applied to many observables. In the present study we show how in this approach the renormalization procedure for the first radiative corrections can be carried out. It requires separating out the contributions which depend on the orientation of the light-front plane. We calculate the electron self-energy as well as the renormalized QED $\gamma e^- \to e^-$ and $\gamma \to e^+e^-$ vertices. Our renormalization procedure is confirmed by recovering, in a straightforward way, the well-known analytical results obtained in the Feynman approach.

1 Introduction

Light-front dynamics [1] is a theoretical approach which has been successfully applied to relativistic composite systems. The two field-theoretical forms of this scheme are: the standard light-front dynamics (LFD) [2] and explicitly covariant light-front dynamics (CLFD) [3]. While the standard LFD deals with the state vector defined on the plane $t+z=0$, this plane is defined in CLFD by the invariant equation $\omega\cdot x=0$, where ω is a four-vector with $\omega^2=0$. The particular choice of the four-vector $\omega=(1,0,0,-1)$ turns CLFD into standard LFD.

In this article, we apply CLFD to the calculation, in first order perturbation theory, of the fermion self-energy and of the QED $\gamma e^- \to e^-$ and $\gamma \to e^+e^-$ vertices. We shall illustrate in detail the calculation technique of CLFD in order to point out its differences and similarities with respect to standard LFD and to the Feynman techniques. As expected from general principles, the LFD amplitude may depend on the orientation of the light-front plane. This dependence manifests itself as a lack of explicit covariance. This is true in particular for off-energy shell amplitudes, or for approximate on-energy shell physical amplitudes. In CLFD this dependence is given in a covariant and well-defined manner, in terms of the four-vector ω. We recall that in LFD, all the particles, even in the intermediate states, are on-mass shell, whereas the amplitude may be off-energy shell.

The question of renormalization in Hamiltonian dynamics has already been addressed a long time ago [4] (see also [5]). The perturbative renormalization in standard LFD has already been done [6] using various methods. In these approaches, it has been shown that the necessary counterterms should be non-local. In our covariant approach this nonlocality manifests itself only in the terms depending on the orientation of the light-front plane. These terms can be explicitly removed in order to calculate the physical amplitude. We will show that after their separation, the renormalization of the ω-independent part of the amplitude is carried out in a very simple way, like in the Feynman approach, and does not require any non-local counterterms. We then find that the on-energy shell electromagnetic vertex in CLFD coincides with the on-mass shell Feynman vertex. The same is true for the electron self-energy. Let us emphasize here that off-energy shell amplitudes in CLFD do not coincide with the corresponding off-mass shell Feynman amplitudes. The coincidence of the LFD amplitudes with the Feynman ones takes place for on-shell amplitudes only. An alternative method of renormalization, using the so-called "minus regularization", has been proposed in [7].

The plan of this article is the following. The renormalized electron mass operator is calculated in Sect. 2. In Sect. 3, the calculation of the corrections of order e^2 to the electron electromagnetic vertex in CLFD is carried out. In Sect. 4 we apply our formalism to the vertex $\gamma \to e^+e^-$ for the threshold value of the photon momentum $Q^2=4m^2$. Section 5 contains our concluding remarks. Some technical details are given in Appendices A and B.

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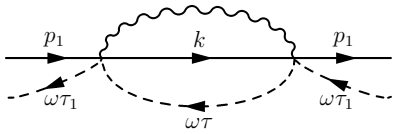

Fig. 1. The electron self-energy graph. The dashed line represents the spurion line, as explained in detail in [3]

2 The electron self-energy

In our explicitly covariant formulation of LFD we are able to follow very closely the standard procedure of renormalization of the fermion self-energy in perturbation theory. The self-energy diagram is shown in Fig. 1.

The general spin structure of the self-energy is very simple. It is given by

$$\Sigma(p,\omega) = A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} + C_1(p^2)\hat{\omega}, \qquad (1)$$

where $\hat{p} = p_\mu\gamma^\mu$ and similarly for $\hat{\omega}$, m is the electron mass, $p = p_1 - \omega\tau_1$ is the total momentum entering the diagram, p_1 is the external fermion momentum, with $p_1^2 = m^2$, and $\omega\tau_1$ is the external spurion momentum. The coefficients A_1, B_1, C_1 are scalar functions of $p^2 = m^2 - 2(\omega{\cdot}p)\tau_1$ only.

The ω-dependent structures should not contribute to observables, like for instance the renormalized mass. In higher order calculation, the term $C_1(p^2)\hat{\omega}$ in (1) may be contracted with other spin structures to give an ω-independent contribution. In the perturbative calculation of the lowest order however, this ω-dependence survives. We introduce therefore the amputated self-energy $\tilde{\Sigma}(p)$ defined by

$$\tilde{\Sigma}(p) = \Sigma(p,\omega) - C_1(p^2)\hat{\omega} = A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m}. \qquad (2)$$

The standard procedure of renormalization of Feynman diagrams relies on two counterterms: the mass counterterm δm^2 and the wave function renormalization proportional to Z_2 [8,9]. Alternatively, and following [10], we can define the renormalized self-energy $\Sigma_{\rm R}(p)$ as the part of $\tilde{\Sigma}(p)$ which is of second order in the variable $(\hat{p} - m)$. Without loss of generality, we can rewrite $\tilde{\Sigma}(p)$ in the form

$$\tilde{\Sigma}(p) = A_0 + (\hat{p} - m)B_0 + \Sigma_{\rm R}(p). \qquad (3)$$

Here A_0, B_0 are constants (they do not depend on p^2), and $\Sigma_{\rm R}(p)$ is the renormalized self-energy written as

$$\Sigma_{\rm R}(p) = (\hat{p} - m)^2\mathcal{M}(p), \qquad (4)$$

where the matrix $\mathcal{M}(p)$ can be represented as

$$\mathcal{M}(p) = a + (\hat{p} + m)b. \qquad (5)$$

The self-energy diagram can easily be iterated to all orders. Supplemented with the adequate contact terms, as indicated on Fig. 2, it leads to the renormalized fermion propagator. This one reads

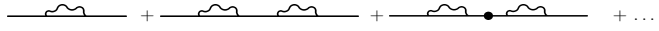

Fig. 2. Iteration of the self-energy contribution, with the corresponding contact interactions (represented by a dot)

$$\frac{1}{\hat{p} - m - (\hat{p} - m)^2\mathcal{M}(p)}.$$

It has the same pole and the same residue at $\hat{p} = m$ as the free propagator of the physical fermion.

The explicit calculation of the renormalized fermion self-energy is now straightforward. According to the rules of CLFD, the electron self-energy shown on Fig. 1 has the form

$$\begin{aligned} -\Sigma(p,\omega) = e^2 \int & \theta(\omega\cdot k)\delta(k^2 - m^2) \\ & \times\gamma^\mu(\hat{k} - \hat{\omega}\tau + m)\gamma^\nu(-g_{\mu\nu})\theta(\omega\cdot(p-k)) \\ & \times\delta((p + \omega\tau - k)^2 - \mu^2)\frac{{\rm d}\tau}{\tau - {\rm i}0}\frac{{\rm d}^4k}{(2\pi)^3} \\ = -\frac{e^2}{(2\pi)^3}\int & \frac{4m - 2\hat{k} + 2\hat{\omega}\tau}{s - p^2}\frac{{\rm d}^2R_\perp {\rm d}x}{2x(1-x)}. \end{aligned} \qquad (6)$$

We introduce here the photon mass μ for infrared regularization. We use for our calculation the Feynman gauge. Since our formulation is explicitly covariant, and we do not rely on the special choice of light-front coordinates, this choice of gauge is of particular interest as compared to the light-cone gauge $A{\cdot}\omega = 0$. It has also the great advantage to enable a direct comparison with the standard textbook result using Feynman diagrams in the Feynman gauge.

The term $\hat{\omega}\tau$ in (6) contributes to $C_1(p^2)\hat{\omega}$ only and can be omitted in the calculation of $\tilde{\Sigma}(p)$. In (6), $\tau = (s - p^2)/(2\omega{\cdot}p)$, and s is the invariant mass of the intermediate photon and electron. It is convenient to introduce the variable $R = k - xp$ with $x = \omega{\cdot}k/\omega{\cdot}p$. As usual (see, e.g., [3]), we represent the spatial part of R as $\boldsymbol{R} = \boldsymbol{R}_\parallel + \boldsymbol{R}_\perp$, where $\boldsymbol{R}_\parallel$ is parallel to $\boldsymbol{\omega}$ and $\boldsymbol{R}_\perp$ is orthogonal to $\boldsymbol{\omega}$. Since, by definition of R, $R{\cdot}\omega = R_0\omega_0 - \boldsymbol{R}_\parallel{\cdot}\boldsymbol{\omega} = 0$, it follows that $R_0 = |\boldsymbol{R}_\parallel|$, and, hence, $\boldsymbol{R}_\perp^2 = -R^2$ is invariant. In this way we find

$$s = \frac{\boldsymbol{R}_\perp^2 + m^2}{x} + \frac{\boldsymbol{R}_\perp^2 + \mu^2}{1 - x}, \quad k{\cdot}p = \frac{R_\perp^2 + m^2}{x} - \frac{1}{2}xp^2,$$

and the phase-space volume is given by ${\rm d}^3k/\varepsilon_k = {\rm d}^2R_\perp\,{\rm d}x/x$.

Now, starting from (6), we can calculate $\Sigma_{\rm R}(p)$ and the scalar coefficients a and b in (5). Knowing $\Sigma(p,\omega)$ from (6), we calculate the coefficients A_1, B_1 in (1) and find $\tilde{\Sigma}(p)$ by (2). Comparing (2) with (3), we express A_0, B_0 through A_1, B_1 for $p^2 = m^2$. Using again the representation (3) for $\tilde{\Sigma}(p)$, we finally obtain the functions a, b, which determine the self-energy (4), through $A_1(p^2), B_1(p^2)$ and A_0, B_0. The details of the calculation are given in Appendix A. For the functions a and b we find

$$a = \frac{\alpha}{4\pi m}\frac{1}{(1-\rho)}\left(1-\frac{2-3\rho}{1-\rho}\log\rho\right),$$
$$b = -\frac{\alpha}{2\pi m^2\rho}\left[\frac{1}{2(1-\rho)}\left(2-\rho+\frac{\rho^2+4\rho-4}{1-\rho}\log\rho\right)+1+\log\frac{\mu^2}{m^2}\right], \quad (7)$$

where

$$\rho = \frac{m^2-p^2}{m^2}.$$

With these expressions for a and b the renormalized mass operator can be written

$$\Sigma_{\rm R}(p) = (\hat{p}-m)^2\mathcal{M}(p) = (\hat{p}-m)^2[a+(\hat{p}+m)b]. \quad (8)$$

It exactly coincides with the standard result given for instance in [8,10], using the same gauge. As already mentioned, the ω-dependent term $C_1(p^2)\hat{\omega}$ may give an ω-independent contribution to the total physical amplitude in higher order calculations, when $\Sigma(p)$ enters as a part of a more complex diagram. It should therefore be considered in that case, and renormalized. This will be the subject of a forthcoming publication dealing with non-perturbative renormalization.

Since the forms of the fermion and antifermion propagators are different in LFD (they contain $(\hat{p}+m)$ for a fermion and $-(\hat{p}-m)$ for an antifermion), the form of the self-energy is also different. However, there is no need to repeat the calculation. The propagators differ by the replacement $\hat{p}\to-\hat{p}$; therefore the renormalized antifermion self-energy $\overline{\Sigma}_{\rm R}(p)$ can be found from the fermion one, (8), by the same replacement:

$$\overline{\Sigma}_{\rm R}(p) = \Sigma_{\rm R}(-p) = (\hat{p}+m)^2[a-(\hat{p}-m)b], \quad (9)$$

where a, b are the same as for the fermion self-energy, and given by (7).

3 The electron electromagnetic vertex

3.1 The anomalous magnetic moment of the electron

The anomalous magnetic moment of the electron is another example of a higher order calculation in QED. Its calculation gives a finite result and does not require renormalization.

The spin 1/2 electromagnetic vertex in CLFD has the general form

$$J_\rho(q) = \bar{u}(p')\Gamma_\rho u(p), \quad (10)$$

where $q = p'-p$. We shall denote $Q^2 = -q^2$. We choose also $\omega.q = 0$, which is always possible for $Q^2 \geq 0$. This implies that the electron scattering process, source of the "external" virtual photon, can be disconnected from the process under investigation [3], at least in the Feynman gauge we are considering in this study. This insures that this external photon can be assimilated to a "physical" particle.

Due to the explicit covariance of our approach, the vertex operator Γ_ρ, according to [11], can be decomposed into

$$\Gamma_\rho = F_1\gamma_\rho + \frac{{\rm i}F_2}{2m}\sigma_{\rho\nu}q^\nu + B_1\left(\frac{\hat{\omega}}{\omega{\cdot}p}-\frac{1}{(1+\eta)m}\right)P_\rho + B_2\frac{m}{\omega{\cdot}p}\omega_\rho + B_3\frac{m^2}{(\omega{\cdot}p)^2}\hat{\omega}\omega_\rho, \quad (11)$$

where $\sigma^{\rho\nu} = {\rm i}(\gamma^\rho\gamma^\nu-\gamma^\nu\gamma^\rho)/2$ and $\eta = Q^2/(4m^2)$. The electromagnetic vertex (10) is gauge invariant since $J_\rho q^\rho = 0$ (with the condition $\omega{\cdot}q = 0$). The possible non-gauge-invariant terms are forbidden by T-invariance. The anomalous magnetic moment is the value of $F_2(Q^2)$ for $Q^2 = 0$.

The physical form factors F_1 and F_2 can easily be extracted from the vertex function Γ_ρ. To this end, we multiply J_ρ by $[\bar{u}^{\sigma'}(p')\gamma^\rho u^\sigma(p)]^*$, $[\bar{u}^{\sigma'}(p'){\rm i}\sigma^{\rho\nu}q_\nu/(2m)u^\sigma(p)]^*$, etc. and sum over polarizations. After taking the trace, we obtain the following quantities:

$$c_1 = {\rm Tr}[O_\rho\gamma^\rho], \quad c_2 = {\rm Tr}[O_\rho{\rm i}\sigma^{\rho\nu}q_\nu]/(2m),$$
$$c_3 = {\rm Tr}[O_\rho(\hat{\omega}/\omega{\cdot}p-1/(1+\eta)m)]P^\rho,$$
$$c_4 = {\rm Tr}[O_\rho]\omega^\rho m/\omega{\cdot}p, \quad c_5 = {\rm Tr}[O_\rho\hat{\omega}]\omega^\rho m^2/(\omega{\cdot}p)^2, \quad (12)$$

where

$$O_\rho = (\hat{p}'+m)\Gamma_\rho(\hat{p}+m)/(4m^2). \quad (13)$$

With the decomposition (11) of Γ_ρ, we get a linear system of five equations for F_1, F_2, B_{1-3} with the inhomogeneous part determined by c_{1-5}. Solving this system relative to F_2, we find

$$F_2 = \frac{1}{4\eta(1+\eta)^2}\Big[(c_3+4c_4-2c_1)(1+\eta)+2(c_1+c_2) -2(c_5+c_4)(1+\eta)^2\Big]. \quad (14)$$

In spite of η in the denominator in (14), there is no singularity at $Q^2 = 0$.

In the usual formulation of LFD on the plane $t+z = 0$, the form factors of spin 1/2 systems are found from the plus-component of the current, i.e., in our notation, from the contraction of J_ρ in (10) with ω_ρ. This contraction is enough to get rid of the contributions proportional to $B_{2,3}$, but not of the term proportional to B_1. The form factors F_1' and F_2' inferred in this way are thus given by

$$J{\cdot}\omega = \bar{u}'\left[F_1\hat{\omega}+\frac{{\rm i}F_2}{2m}\sigma_{\rho\nu}\omega^\rho q^\nu+2B_1\left(\hat{\omega}-\frac{\omega{\cdot}p}{(1+\eta)m}\right)\right]u \equiv \bar{u}'\left[F_1'\gamma_\rho+\frac{{\rm i}F_2'}{2m}\sigma_{\rho\nu}q^\nu\right]u\,\omega^\rho, \quad (15)$$

where

$$F_1' = F_1+\frac{2\eta B_1}{1+\eta}, \quad F_2' = F_2+\frac{2}{1+\eta}B_1. \quad (16)$$

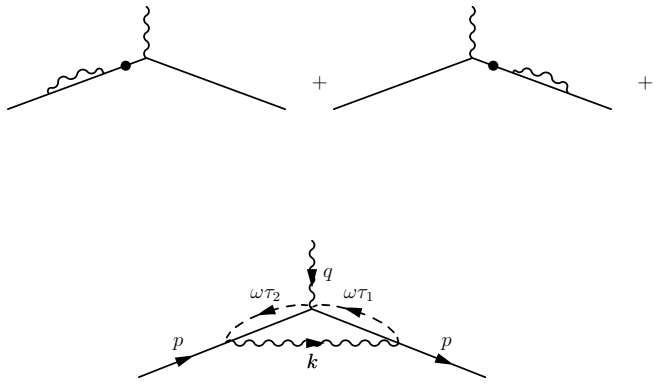

Fig. 3. Radiative corrections to the electromagnetic vertex of the electron. For simplicity, we have not indicated, in the first two diagrams, the spurion lines and the momenta

The B_1 expression can be found from the above mentioned system of equations, leading to

$$B_1 = -\frac{1}{8\eta(1+\eta)}\Big[(c_3+4c_4-2c_1)(1+\eta) +2(c_1+c_2)-4c_5(1+\eta)^2\Big]. \quad (17)$$

Substituting F_2 from (14) and B_1 from (17) into (16) for F_2', we get

$$F_2' = (c_5-c_4)/(2\eta). \quad (18)$$

Of course, in a given order of perturbation theory, both methods for calculating the form factors, by (14) and (18), should give the same result. This means we should find $B_1 = 0$. We shall see below that this is indeed the case.

Let us first calculate the form factor F_2', for $Q^2 = 0$. The radiative corrections to the electromagnetic vertex are composed of three irreducible contributions shown on Fig. 3. The first two diagrams involve the renormalized self-energy calculated in Sect. 2. For on-energy shell external electrons, they result in the renormalized electron mass, as usual. The third one can easily be calculated according to the rules of the graph techniques [3]. It has the form

$$\begin{aligned}\bar u(p)\Gamma^\rho u(p) = e^2\int \bar u(p)\gamma_\mu(\hat p-\hat k+m)\theta(\omega\cdot(p-k))\\ \times\delta((p+\omega\tau_1-k)^2-m^2)\frac{\mathrm{d}\tau_1}{\tau_1-\mathrm{i}0}\\ \times\gamma^\rho(\hat p-\hat k+m)\theta(\omega\cdot(p-k))\\ \times\delta((p+\omega\tau_2-k)^2-m^2)\frac{\mathrm{d}\tau_2}{\tau_2-\mathrm{i}0}\gamma_\nu u(p)\\ \times(-g^{\mu\nu})\theta(\omega\cdot k)\delta(k^2-\mu^2)\frac{\mathrm{d}^4k}{(2\pi)^3}. \quad (19)\end{aligned}$$

The factor $\hat p-\hat k = \hat k_1-\hat\omega\tau_1 = \hat k_2-\hat\omega\tau_2$ includes the contact terms $-\hat\omega\tau_1$ and $-\hat\omega\tau_2$, as explained in [3]. For the regularization of subsequent calculations, we introduced in (19) the photon mass μ, although it is not necessary in the present subsection.

Integrating over τ_1, τ_2 and k_0, we get

$$\bar u(p)\Gamma^\rho u(p) = e^2\int\frac{\bar u(p)G^\rho u(p)}{(s-m^2)^2(1-x)^2}\frac{\mathrm{d}^3k}{2\varepsilon_k(2\pi)^3}, \quad (20)$$

where $s=(k+k_1)^2=(k+k_2)^2$, $x=\omega\cdot k/\omega\cdot p$ and we note

$$G^\rho = -\gamma_\mu(\hat p-\hat k+m)\gamma^\rho(\hat p-\hat k+m)\gamma^\mu. \quad (21)$$

The integrands for the scalar functions c_{1-5} are represented in terms of the scalar products between the four-momenta p, k and ω. The scalar product $p\cdot k$ is given by

$$p\cdot k = \mu^2/2+(1-x)(s-m^2)/2,$$

whereas the scalar products $\omega\cdot k$ and $\omega\cdot p$ always appear in the ratio x, with $0\le x\le 1$.

In terms of $\boldsymbol{R}_\perp^2$ and x, the variable s can be written

$$s = \frac{\boldsymbol{R}_\perp^2+\mu^2}{x}+\frac{\boldsymbol{R}_\perp^2+m^2}{1-x}. \quad (22)$$

Substituting these expressions into (20), we find (for $\mu = 0$)

$$\bar u(p)\Gamma^\rho u(p) = \frac{\alpha}{4\pi^2}\int\bar u(p)G^\rho u(p)\frac{x\mathrm{d}x\mathrm{d}^2R_\perp}{(\boldsymbol{R}_\perp^2+xm^2)^2}, \quad (23)$$

where we denote $\alpha = e^2/4\pi$. To calculate F_2' by (18) (for $\eta\to 0$), we substitute Γ_ρ from (23) into (13) (for $p'=p$) and then into expressions (12) for c_4 and c_5. Calculating the traces, we get

$$F_2'(0) = \frac{\alpha}{4\pi^2}\int 4m^2x(1-x)\frac{x\mathrm{d}x\mathrm{d}^2R_\perp}{(\boldsymbol{R}_\perp^2+xm^2)^2}. \quad (24)$$

We thus obtain the well-known result for the anomalous magnetic moment of the electron:

$$F_2'(0) = \frac{\alpha}{2\pi}. \quad (25)$$

Now consider the form factor F_2 calculated after separation of the ω-dependent terms. According to (16), it is related to F_2' by $F_2(0) = F_2'(0) - 2B_1(0)$. From (17), for $Q^2=0$, we find the following expression for B_1:

$$\begin{aligned}B_1(0) = \frac{\alpha}{2\pi}\int\frac{[m^2(2-x)x^2-2\boldsymbol{R}_\perp^2(1-x)-\mu^2(2-x)]}{[\boldsymbol{R}_\perp^2+m^2x^2+\mu^2(1-x)]^2}\\ \times R_\perp\mathrm{d}R_\perp\mathrm{d}x, \quad (26)\end{aligned}$$

which is logarithmically divergent. We regularize it using the Pauli–Villars method, i.e., the photon propagator is replaced by

$$\frac{1}{k^2-\mu^2}\to\frac{1}{k^2-\mu^2}-\frac{1}{k^2-\Lambda^2}. \quad (27)$$

In the absence of an infrared singularity we can put in (27) $\mu = 0$. Hence, the regularized expression for B_1 reads

$$B_1^{\mathrm{reg}} = B_1(\mu=0)-B_1(\mu=\Lambda).$$

Integrating over $R_\perp$, we get

$$B_1^{\rm reg} = -\frac{\alpha}{4\pi} \times \int_0^1 {\rm d}x \frac{\rm d}{{\rm d}x}\left[x(2-x)\log\left(\frac{\Lambda^2(1-x)+m^2x^2}{m^2x^2}\right)\right]. \quad (28)$$

After integration over x we get $B_1^{\rm reg} = 0$ for any value of Λ. This clearly shows that both methods to calculate the anomalous magnetic moment of the electron give the same result.

3.2 The renormalized electron charge

In order to calculate the radiative correction to the form factor F_1, we have to renormalize the charge. The renormalization means that the Lagrangian contains a counterterm of the form

$$Z_1\bar\psi\gamma^\rho\psi A_\rho;$$

hence, the amplitude J^ρ is replaced by

$$J^\rho \to J^\rho_{\rm ren} = J^\rho - J^\rho_0, \quad (29)$$

where

$$J^\rho_0 = Z_1\bar u(p')\gamma^\rho u(p).$$

The renormalization procedure is described in many textbooks, see for example [8–10]. In order to find Z_1, one must calculate the amplitude $\bar u(p)\Gamma^\rho u(p)$ from the diagram of Fig. 3. The value of Z_1 is in fact just the form factor $F_1(0)$ determined by this diagram. For $p = p'$ the general decomposition (10) turns into

$$\bar u(p)\Gamma^\rho u(p) = Z_1\bar u(p)\gamma^\rho u(p) + Z'\frac{\omega^\rho m}{\omega{\cdot}p}\bar u(p)u(p), \quad (30)$$

where $Z_1 = F_1(0)$ and $Z' = B_2(0) + B_3(0)$.

From (30) the constant Z_1 is given by

$$Z_1 = \frac{1}{4\omega{\cdot}p}{\rm Tr}\left[\omega_\rho\Gamma^\rho(\hat p + m)\right]. \quad (31)$$

The vertex Γ^ρ is determined by (19) and is reduced to (20). For regularization purposes, we should now keep the photon mass μ finite. From (31) we find

$$Z_1 = \frac{\alpha}{(2\pi)^3}\int {\rm d}^2R_\perp \int_0^1 \frac{\left[\boldsymbol{R}_\perp^2 + m^2(-2+2x+x^2)\right]x}{\left[\boldsymbol{R}_\perp^2 + m^2x^2 + \mu^2(1-x)\right]^2}{\rm d}x. \quad (32)$$

The subsequent calculation is straightforward. We calculate $Z_1(\mu, L)$ for a fixed upper limit L of the variable $R_\perp$ in the integral (32), take the difference $Z_1(\mu, L) - Z_1(\Lambda, L)$, take the limit $L \to \infty$ and then calculate the limits $\mu \to 0$ and $\Lambda \to \infty$. We then obtain

$$Z_1(\mu\to 0, \Lambda\to\infty) = \frac{9\alpha}{8\pi} + \frac{\alpha}{2\pi}\log\left(\frac{\mu^2}{m^2}\right) + \frac{\alpha}{4\pi}\log\left(\frac{\Lambda^2}{m^2}\right). \quad (33)$$

This expression exactly coincides with the expression found in the Feynman formalism [12]. We emphasize that this result for Z_1 is obtained for the physical part of the full vertex (30), after separating out the unphysical term proportional to $Z'\omega_\rho$. The latter term can be disregarded; there is no need to calculate it.

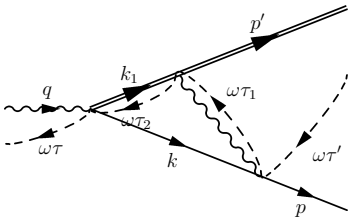

Fig. 4. Light-front time ordered graph for $\gamma \to e^+e^-$. A similar diagram with the opposite time ordering for the photon exchange should be added

4 Application to the vertex $\gamma \to e^+e^-$

As a direct application of the preceding calculation, let us now consider the electromagnetic transition $\gamma \to e^+e^-$, when the photon is off-energy shell. This vertex is of particular interest since it can directly be used in the calculation of the decay width of orthomuonium into e^+e^- (with the replacement of the initial e^+e^--pair by $\mu^+\mu^-$). In the Weisskopf–Van Royen limit, the decay width is proportional to the elementary vertex $\mu^+\mu^- \to \gamma$ (or, equivalently, to $\gamma \to \mu^+\mu^-$), where the $\mu^+\mu^-$-pair originates from the muonium wave function with zero relative momentum, i.e. with $p_{\mu^+} = p_{\mu^-} = p$. This process has also a direct application in hadronic physics, for the leptonic decay width of charmonium states [13,14].

4.1 Spin structure

The amplitude of the process $\gamma \to e^+e^-$ is shown in Fig. 4. We consider below this amplitude for the e^+e^- state on-energy shell, i.e., with $\tau' = 0$ in Fig. 4. Moreover, we take the e^+e^- c.m. energy equal to $2m$, which corresponds to $Q^2 = (q - \omega\tau)^2 = -2(q{\cdot}\omega)\tau = 4m^2$. The incoming photon is thus off-energy shell, as it is for the electromagnetic form factors discussed in Sect. 3.

The amplitude for this process depends on the four-vectors p and ω. Its general structure thus reads

$$\bar u(p)M^\rho v(p) = A\bar u(p)\gamma^\rho v(p) + B\frac{p^\rho}{\omega{\cdot}p}\bar u(p)\hat\omega v(p) + C\frac{\omega^\rho m^2}{(\omega{\cdot}p)^2}\bar u(p)\hat\omega v(p). \quad (34)$$

Here $v(p)$ is the positron spinor. The constant A in (34) is the value of the form factor $F_1(Q^2)$ at $Q^2 = 4m^2$. One

can also construct the structure $\sigma^{\rho\beta}\omega_\beta/\omega{\cdot}p$, but it is not independent, since

$$\frac{\mathrm{i}m}{\omega{\cdot}p}\bar u(p)\sigma^{\rho\beta}\omega_\beta v(p)=\bar u(p)\gamma^\rho v(p)-\frac{p^\rho}{\omega{\cdot}p}\bar u(p)\hat\omega v(p).$$

Multiplying (34) on the left by $u(p)$ and on the right by $\bar v(p)$ and summing over polarizations, we get the factors $\sum_\lambda u(p)\bar u(p)=(\hat p+m)$, $\sum_\lambda v(p)\bar v(p)=(\hat p-m)$. We introduce therefore the quantity

$$\tilde M^\rho=(\hat p+m)M^\rho(\hat p-m), \tag{35}$$

and calculate the following traces:

$$\begin{aligned}
T_1&\equiv\frac{1}{16m^2}\mathrm{Tr}\left[\tilde M^\rho\gamma_\rho\right]=(-3A+C)/2,\\
T_2&\equiv\frac{1}{16m^2}\mathrm{Tr}\left[\tilde M^\rho\hat\omega\right]\frac{p_\rho}{\omega{\cdot}p}=(B+C)/2,\\
T_3&\equiv\frac{1}{16m^2}\mathrm{Tr}\left[\tilde M^\rho\hat\omega\right]\frac{\omega_\rho m^2}{(\omega{\cdot}p)^2}=(A+B)/2.
\end{aligned}\tag{36}$$

We can thus find the coefficients which determine the amplitude (34):

$$\begin{aligned}
A&=T_2-T_1-T_3,\\
B&=T_1-T_2+3T_3,\\
C&=-T_1+3T_2-3T_3.
\end{aligned}\tag{37}$$

Below we will call A the "physical amplitude", in analogy with the form factor calculation of Sect. 3.

4.2 The physical amplitude A

The amplitude corresponding to the diagram of Fig. 4 is given by the rules of the graph techniques [3] and writes

$$\begin{aligned}
\bar u(p)M_1^\rho v(p')=e^2\int&\bar u(p)\gamma_\mu(\hat k+m)\theta(\omega{\cdot}k)\\
&\times\delta(k^2-m^2)\frac{\mathrm{d}^4k}{(2\pi)^3}\\
&\times\gamma^\rho\left(m-(\hat Q-\hat k)\right)\theta(\omega{\cdot}(Q-k))\\
&\times\delta\left((Q-k+\omega\tau_2)^2-m^2\right)\frac{\mathrm{d}\tau_2}{\tau_2-\mathrm{i}0}\\
&\times\gamma_\nu v(p')(-g^{\mu\nu})\\
&\times\delta\left((p-\omega\tau'+\omega\tau_1-k)^2-\mu^2\right)\\
&\times\theta(\omega{\cdot}(p-k))\frac{\mathrm{d}\tau_1}{\tau_1-\mathrm{i}0}.
\end{aligned}\tag{38}$$

where $Q=q-\omega\tau$. Note that the fermion and antifermion propagators in LFD differ from each other. The propagator $(\hat k+m)$ in (38) corresponds to the electron, whereas the propagator $\left(m-(\hat Q-\hat k)\right)$ corresponds to the positron.

The factor $m-(\hat Q-\hat k)=m-(\hat k_1-\hat\omega\tau_2)$ incorporates the difference $\hat k_1-\hat\omega\tau_2$ and, therefore, takes into account the contact term $-\hat\omega\tau_2$. We consider in this section the case $Q^2=4m^2$, relevant for the decay width of the positronium, so that $p'=p$, $\tau'=0$ and so the two graphs corresponding to the two different time orderings should give the same contribution $M^\rho=M_1^\rho+M_2^\rho=2M_1^\rho$.

After integration over τ_1 and τ_2, the amplitude (38) is given by

$$\begin{aligned}
&\bar u(p)M_1^\rho v(p)\\
&\quad=-e^2\int\bar u(p)\gamma_\mu(\hat k+m)\gamma^\rho\left(m-(\hat Q-\hat k)\right)\gamma^\mu v(p)\\
&\quad\times\frac{\theta(\omega{\cdot}(p-k))\theta(\omega{\cdot}(Q-k))\theta(\omega{\cdot}k)}{(s_{12}-Q^2)\left(1-\dfrac{\omega{\cdot}k}{\omega{\cdot}Q}\right)\,(s_{123}-Q^2)\left(\dfrac{\omega{\cdot}p-\omega{\cdot}k}{\omega{\cdot}Q}\right)}\\
&\quad\times\frac{\mathrm{d}^3k}{2\varepsilon_k(2\pi)^3},
\end{aligned}\tag{39}$$

where

$$s_{12}-Q^2=2(\omega{\cdot}Q)\tau_2,\quad s_{123}-Q^2=2(\omega{\cdot}Q)\tau_1,$$

and

$$\begin{aligned}
s_{12}&=(k+k_1)^2=\frac{\boldsymbol{R}_{k\perp}^2+m^2}{x_k}+\frac{\boldsymbol{R}_{k\perp}^2+m^2}{1-x_k},\\
s_{123}&=(k+k_2+p)^2\\
&=\frac{\boldsymbol{R}_{k\perp}^2+m^2}{x_k}+\frac{\boldsymbol{R}_{k_1\perp}^2+\mu^2}{x_{k_1}}+\frac{\boldsymbol{R}_{p'\perp}^2+m^2}{x_{p'}}\\
&=\frac{\boldsymbol{R}_{k\perp}^2+m^2}{x_k}+\frac{\boldsymbol{R}_{k\perp}^2+\mu^2}{1/2-x_k}+2m^2.
\end{aligned}\tag{40}$$

Like in the previous sections, we define above the variables $R_l=l-x_lQ$ with $x_l=\omega{\cdot}l/\omega{\cdot}Q$, where l is either k,k_1 or p'. At the threshold $Q^2=4m^2$, we have $\boldsymbol{R}_{p'\perp}=0$ and $x_{p'}=1/2$ in the variable s_{123}. We thus find

$$\tilde M^\rho=-2\frac{e^2}{(2\pi)^3}\int\mathrm{d}^2R_\perp \tag{41}$$

$$\times\int_0^{1/2}\frac{O^\rho}{(s_{12}-4m^2)(1-x)(s_{123}-4m^2)\,(1/2-x)}\times\frac{\mathrm{d}x}{2x},$$

where

$$O^\rho=(\hat p+m)\gamma_\mu(\hat k+m)\gamma^\rho\left(m-(\hat Q-\hat k)\right)\gamma^\mu(\hat p-m). \tag{42}$$

The amplitude $\tilde M^\rho$ is connected to M^ρ by (35). The factor 2 in (42) results from the sum of the two amplitudes M_1 and M_2. In order to find the coefficients A,B,C which determine the amplitude (34), we substitute $\tilde M^\rho$ into (36), (37), regularize the expression by the Pauli–Villars prescription

$$A\to A(\mu)-A(\Lambda),$$

(and similarly for B,C), and take the limits $\mu\to0$, $\Lambda\to\infty$. The details of the calculation are given in Appendix B. The final expression for A is then

$$A=\frac{\alpha m}{\mu}-\frac{7\alpha}{8\pi}+\frac{\alpha}{2\pi}\log\left(\frac{\mu^2}{m^2}\right)+\frac{\alpha}{4\pi}\log\left(\frac{\Lambda^2}{m^2}\right). \tag{43}$$

This expression exactly coincides with that calculated in the Feynman formalism [12].

The integral for $B(\mu)$ converges and does not depend on μ:

$$B(\mu) = -\frac{\alpha}{4\pi}. \tag{44}$$

The amplitude regularized à la Pauli–Villars is determined by the difference $B_{\rm reg} = B(\mu) - B(\Lambda)$. It is therefore zero.

From (43) and (33) we recover the renormalized amplitude

$$A_{\rm ren} = A - Z = \frac{\alpha m}{\mu} - \frac{2\alpha}{\pi}, \tag{45}$$

which coincides with the result found in the Feynman approach [12]. It contains the term $\alpha m/\mu$, corresponding to an infrared singularity. This infrared singularity is well known. For the calculation of the radiative correction to the leptonic decay width of charmonium states for instance, it is properly accounted for when the Coulomb interaction which gives rise to the two-body wave function is taken care of [12,14].

The calculation of C gives a divergent result even after a single Pauli–Villars regularization. However, C is the coefficient in front of the term $\omega^\rho \bar{u}\hat{\omega}u$ which is proportional to ω^ρ.

In all perturbative processes where the e^+e^--pair is on-energy shell, the amplitude A gives, by definition, the physical amplitude. For the leptonic decay width of charmonium states [13] in the Weisskopf–Van Royen approximation we considered in this chapter, one can easily check that, according to (15)–(23) in this reference, a vertex factor of the form (34) contributes only through the A term. The calculation of the decay width beyond this approximation will be investigated in a forthcoming publication [14].

5 Conclusion

The understanding of perturbative renormalization in QED is an unavoidable step before studying more subtle systems like QCD. While this perturbative renormalization is now a textbook section for the standard formulation of field theory using Feynman graph techniques, it is not as well understood in light-front quantization. The main reason is the difficulty to exhibit the covariant structure of the electromagnetic vertex and electron self-energy since standard LFD explicitly breaks covariance.

We have shown in this study that the covariant formulation of LFD is a powerful tool to make the link between LFD and Feynman approaches. The explicit covariance of our formulation enables us to exhibit the relativistic structure of the electron self-energy as well as the electromagnetic vertex in QED. We are thus in a position to extract, after renormalization, the finite physical contribution from the infinite amplitude. To do that, we have to know the dependence of the amplitudes on the orientation, ω, of the light front. This is trivial in CLFD. In the standard formulation of LFD, this dependence cannot always be disentangled from the physical part of the amplitude.

The finite physical amplitude we found in our approach for the electron self-energy, the electromagnetic vertex, and the $\gamma \to e^+e^-$ amplitude agree thus with the standard textbook results. This agreement, in the CLFD approach, was obtained using the Feynman gauge and, thus, confirms the possibility to use this gauge in our covariant formulation. Though this agreement is, of course, expected for the correct calculation in any gauge, it is not trivial, since the renormalization procedure in CLFD necessitates the separation of ω-dependent structures in the decomposition of the general amplitude, as given for instance in (11) and (34). These structures are absent in the Feynman approach. The agreement of both calculations is a strong test of our procedure. It allows us to safely apply our procedure to the light-front off-energy shell amplitudes, which do not coincide with the Feynman amplitudes.

We emphasize that in order to reproduce these results, a covariant regularization of divergences (Pauli–Villars in the present study) is important. The attempt to regularize the integrals by a cut-off in the variable $R_\perp$, for instance, allows us of course to work with finite integrals. It gives finite, but wrong, renormalized results.

In the present study, we were interested in the renormalization of the fermion self-energy and the electromagnetic vertex only and therefore we did not analyze the vacuum polarization contribution (photon self-energy), keeping also in mind an application to the calculation of the relativistic gluon radiative correction to the J/ψ leptonic decay width.

The renormalization of off-energy shell vertex is applied to the latter process in [14]. The QED vertex $e^+e^- \to \gamma$, corrected by a color factor, coincides with the QCD vertex $q^+q^- \to \gamma$. This calculation, beyond the non-relativistic approximation, requires the renormalization of the amplitude $q^+q^- \to \gamma$, which is off-energy shell not only in the photon leg, but also in the q^+q^- state. The vacuum polarization is in that case an electromagnetic correction to this strong process and can therefore be neglected.

We have considered in this study a few examples of the first order perturbative renormalization in CLFD. These examples, as well as a successful application of CLFD to the analysis of the cutoff dependence of the binding energy in the Yukawa model [15], give an indication that the formalism of CLFD can be useful for non-perturbative studies too. The question of non-perturbative renormalization for scalar particles, will be addressed in a forthcoming publication [16].

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Appendix

A Calculation of a and b

According to (4) and (5), the self-energy $\Sigma_{\rm R}(p)$ is determined by the scalar functions a and b. As we will see below, a and b are determined by the coefficients $A_1(p^2)$ and $B_1(p^2)$ in the general decomposition (1) and by their combinations in the limit $p^2 \to m^2$.

From (1) we find the coefficients $A_1(p^2)$ and $B_1(p^2)$:

$$A_1 = \frac{1}{4}{\rm Tr}[\Sigma(p)], \quad B_1 = \frac{m}{4\omega{\cdot}p}{\rm Tr}[\Sigma(p)\hat{\omega}]. \tag{46}$$

Substituting here (6) for $\Sigma(p)$ we get

$$A_1(p^2) = \frac{\alpha m}{\pi^2} \times \int \frac{\pi {\rm d}R_\perp^2 \,{\rm d}x}{R_\perp^2 + (1-x)m^2 + x[\mu^2 + (1-x)p^2]}, \tag{47}$$

$$B_1(p^2) = -\frac{\alpha m}{2\pi^2} \times \int \frac{\pi {\rm d}R_\perp^2 \,x{\rm d}x}{R_\perp^2 + (1-x)m^2 + x[\mu^2 + (1-x)p^2]}. \tag{48}$$

These integrals diverge logarithmically.

Comparing (1) with (3) and taking into account (4), we find

$$A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} = A_0 + (\hat{p}-m)B_0 + (\hat{p}-m)^2\mathcal{M}(p). \tag{49}$$

From here we can express the constants A_0 and B_0 through A_1 and B_1:

$$A_0 = \frac{1}{4m}{\rm Tr}\left[\left(A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m}\right)(\hat{p}+m)\right]_{p^2=m^2} = A_1(m^2) + B_1(m^2) \tag{50}$$

$$B_0 = \frac{1}{4m(p^2-m^2)} \times {\rm Tr}\left[\left(A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} - A_0\right)(\hat{p}+m)^2\right]_{p^2\to m^2}. \tag{51}$$

We thus obtain

$$A_0 = \frac{\alpha m}{2\pi^2}\int \frac{(2-x)\pi {\rm d}R_\perp^2 \,{\rm d}x}{R_\perp^2 + (1-x)^2m^2 + x\mu^2}, \tag{52}$$

$$B_0 = -\frac{\alpha}{2\pi^2}\int \frac{x[R_\perp^2 - (3-4x+x^2)m^2 + x\mu^2]\pi {\rm d}R_\perp^2 \,{\rm d}x}{[R_\perp^2 + (1-x)^2m^2 + x\mu^2]^2}. \tag{53}$$

These integrals also diverge logarithmically.

From (49), and taking into account (5) for $\mathcal{M}(p)$, we get

$$(\hat{p}-m)^2(a + (\hat{p}+m)b) = A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} - A_0 - (\hat{p}-m)B_0.$$

This allows us to find a and b:

$$a = \frac{1}{4p^2(p^2-m^2)}{\rm Tr}\left[\left(A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} - A_0 - (\hat{p}-m)B_0\right)(\hat{p}+m)\hat{p}\right] = \frac{A_1(p^2) + B_1(p^2) - A_0}{p^2-m^2},$$

$$b = \frac{1}{4p^2(p^2-m^2)^2}{\rm Tr}\left[\left(A_1(p^2) + B_1(p^2)\frac{\hat{p}}{m} - A_0 - (\hat{p}-m)B_0\right)(\hat{p}+m)^2\hat{p}\right] = \frac{2m(A_1(p^2)-A_0)}{(p^2-m^2)^2} + \frac{(p^2+m^2)B_1(p^2)}{m(p^2-m^2)^2} - \frac{B_0}{p^2-m^2}. \tag{54}$$

Substituting here the above expressions for A_1, B_1, A_0 and B_0, we get

$$a = \frac{\alpha m}{2\pi}\int \frac{x(2-3x+x^2)}{[R_\perp^2 + m^2(1-x)^2]} \times \frac{{\rm d}R_\perp^2 \,{\rm d}x}{[R_\perp^2 + m^2(1-x)(1-(1-\rho)x)]}. \tag{55}$$

$$b = -\frac{\alpha}{2\pi}\int \frac{x^2(1-x)}{[R_\perp^2 + m^2(1-x)^2 + \mu^2x]^2} \times \frac{[R_\perp^2 - m^2(3-4x+x^2)]{\rm d}R_\perp^2 \,{\rm d}x}{[R_\perp^2 + m^2(1-x)(1-(1-\rho)x) + \mu^2x]}. \tag{56}$$

We omitted in a the photon mass μ, since that integral has no infrared divergence, and introduced the notation $\rho = (m^2-p^2)/m^2$. Integrating over $R_\perp^2$ and x and keeping in b the leading term in $\log(\mu^2/m^2)$ only, we obtain (7).

One can similarly calculate the coefficient C_1 determining the ω-dependent part of $\Sigma(p)$. It is given by

$$C_1(p^2) = \frac{1}{4\omega{\cdot}p}{\rm Tr}\left[\Sigma(p)\left(\hat{p} - \frac{p^2\hat{\omega}}{\omega{\cdot}p}\right)\right] = -\frac{\alpha}{4\pi^2\omega{\cdot}p} \times \int \frac{[2R_\perp^2 + m^2(2-3(1-\rho)x^2)]\pi {\rm d}R_\perp^2 \,{\rm d}x}{[R_\perp^2 + m^2(1-x)(1-(1-\rho)x) + \mu^2x]x}. \tag{57}$$

It is quadratically divergent in the variable $R_\perp$ and is logarithmically divergent at $x = 0$. Note that the standard Pauli–Villars regularization is not enough to make it finite.

B Calculation of A, B and C

As explained in Sect. 4.2, we substitute $\tilde{M}^\rho$ into (36), (37) in order to find the coefficients A, B, C determining the amplitude (34). We thus find

$$(A,B,C) = -\frac{8\pi\alpha}{(2\pi)^3}\int \mathrm{d}^2R_\perp \times\int_0^{1/2}\frac{(a,b,c)}{(s_{12}-4m^2)(1-x)(s_{123}-4m^2)(1/2-x)} \times\frac{\mathrm{d}x}{2x}, \tag{58}$$

with

$$\begin{aligned} a &= t_2 - t_1 - t_3 \\ &= \frac{1}{16m^2}\left\{\mathrm{Tr}\left[O^\rho\hat{\omega}\right]\frac{p_\rho}{\omega{\cdot}p} - \mathrm{Tr}\left[O^\rho\gamma_\rho\right] - \mathrm{Tr}\left[\tilde{M}^\rho\hat{\omega}\right]\frac{\omega_\rho m^2}{(\omega{\cdot}p)^2}\right\} \\ &= -\frac{1}{x}\left[\boldsymbol{R}_\perp^2(1-2x)+m^2(1+4x^2)\right], \\ b &= t_1 - t_2 + 3t_3 \\ &= \frac{1}{x}\left[\boldsymbol{R}_\perp^2(1-4x)+m^2(1-2x)^2(1+2x)\right], \\ c &= -t_1 + 3t_2 - 3t_3 \\ &= -\frac{1}{4m^2x^2}\left[\boldsymbol{R}_\perp^4+2m^2\boldsymbol{R}_\perp^2(1-8x^2)+m^4(1-4x^2)^2\right]. \end{aligned} \tag{59}$$

To calculate the traces (59), we need the following scalar products:

$$\begin{aligned} &k{\cdot}Q = 2m^2 + (1-x)(s_{12}-4m^2), \quad k{\cdot}p = k{\cdot}Q/2, \\ &p{\cdot}Q = Q^2/2 = 2m^2. \end{aligned} \tag{60}$$

Substituting (59) into (58), we find

$$A(\mu) = \frac{8\pi\alpha}{(2\pi)^3}\int \mathrm{d}^2R_\perp \times\int_0^{1/2}\frac{\left[\boldsymbol{R}_\perp^2(1-2x)+m^2(1+4x^2)\right]}{[\boldsymbol{R}_\perp^2+m^2(1-2x)^2]} \times\frac{\mathrm{d}x}{[\boldsymbol{R}_\perp^2+m^2(1-2x)^2+2\mu^2x]}, \tag{61}$$

$$B(\mu) = -\frac{8\pi\alpha}{(2\pi)^3}\int \mathrm{d}^2R_\perp \times\int_0^{1/2}\frac{\left[\boldsymbol{R}_\perp^2(1-4x)+m^2(1-2x)^2(1+2x)\right]}{[\boldsymbol{R}_\perp^2+m^2(1-2x)^2]} \times\frac{\mathrm{d}x}{[\boldsymbol{R}_\perp^2+m^2(1-2x)^2+2\mu^2x]}, \tag{62}$$

$$C(\mu) = \frac{4\pi\alpha}{(2\pi)^3}\int \mathrm{d}^2R_\perp \times\int_0^{1/2}\frac{\left[\boldsymbol{R}_\perp^4+2m^2\boldsymbol{R}_\perp^2(1-8x^2)+m^4(1-4x^2)^2\right]}{2m^2x\,[\boldsymbol{R}_\perp^2+m^2(1-2x)^2]} \times\frac{\mathrm{d}x}{[\boldsymbol{R}_\perp^2+m^2(1-2x)^2+2\mu^2x]}. \tag{63}$$

The integral (63) for C, which is the coefficient in front of the structure proportional to ω_ρ, diverges quadratically at $R_\perp\to\infty$ and logarithmically at $x=0$. The integral (61) for A logarithmically diverges at $R_\perp\to\infty$. The integral (62) for B at $R_\perp\to\infty$ has the asymptotic expression

$$B(\mu) \propto \int_0^\infty \frac{\mathrm{d}R_\perp}{R_\perp}\int_0^{1/2}(1-4x)\mathrm{d}x.$$

Since the integral over x is zero, $B(\mu)$ is finite.

The regularization of $A(\mu)$ proceeds as follows. The integral over x in $A(\mu)$ can be done analytically. In the Pauli–Villars regularization scheme, we should take the difference $A(\mu)-A(\Lambda)$ and calculate the convergent integral over $R_\perp$. Equivalently, but technically easier, we calculate $A(\mu,L)$ with the cutoff L in the variable $R_\perp$, take the difference $A(\mu,L)-A(\Lambda,L)$ and then take the limit $L\to\infty$. The result is analytic, but lengthy. In the limits $\mu\to 0$ and $\Lambda\to\infty$ we obtain (43), which coincides with the result calculated in the Feynman formalism [12].

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